One-Step Computational Method for the Determination of Charge on Capacitors using the Hermite Polynomial

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ABSTRACT

Description of circuits using differential equations is very convenient for the electrical circuits' behavioral analysis. In this paper, a one-step fifth-order computational method is proposed for the solution of second order differential equations using the Hermite polynomial as a basis function. The computational method was then applied on two real-life problems in physics to determine the charge on the capacitors and from the results obtained, it is obvious that the method is computationally reliable. The basic properties of the method were further investigated and found to be zero-stable, consistent and convergent.

KEYWORDS: Capacitors, Charge, Computational Method, Hermite Polynomial, One-Step Method
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Introduction

A number of problems in science and technology can be formulated into differential equations. The analytical methods of solving differential equations are applicable to a limited class of equations. Quite often differential equations appearing in physical problems do not have exact solutions and one is obliged to resort to computational methods to solve such problems.

In this paper, a one-step computational method for the determination of charge on capacitors occurring in the form of second order differential equations of the form shall be proposed,

$$y'' = f(x, y, y'),\ y(x_0) = y_0,\ y'(x_0) = y_0' \qquad (1)$$

where f is continuous within the interval of integration.

Direct methods for the solution of higher-order Ordinary Differential Equations (ODEs) have been proposed by many authors and they concluded that direct methods are more convenient and accurate than the method of reduction to systems of first order ODEs (Awoyemi, 2008). Some of the authors that proposed direct methods include Adesanya *et al.,* (2008), Awoyemi (2001), to mention a few. These authors proposed continuous implicit linear multistep methods which were implemented in predictor-corrector mode where they developed reducing order predictors to implement the corrector. Adesanya *et al.,* (2012) reported that one of the setbacks of predictor-corrector method is that it is very costly to implement as subroutine are very complicated to write because it requires special technique to supply the starting values and varying step size leads to longer computer time and human efforts. Above all, the predictors are in reducing order; hence it affects the accuracy of the method. Awoyemi (1999) reported that continuous linear multistep method has greater advantages over the discrete method in that it gives better error estimation, provide a simplified coefficient for further analytical work at different points and guarantee easy approximation of solution at all interior points within the interval of integration.

Scholars later developed block methods to cater for some of the setbacks of predictor-corrector methods mentioned above. Block method generates independent solution at selected grid point without overlapping. It is less expensive in terms of the number of function evaluation compared to predictor-corrector method; moreover, it possesses the properties of Runge-Kutta method for being self-starting and does not require starting values. Some of the authors that proposed block methods using different approximate solutions are Awoyemi (2008), Anake *et al.*, (2012), Owolabi (2012), James *et al.*, (2013), Sunday *et al.*, (2013), Adesanya *et al.*, (2014), Sunday *et al.*, (2014a), Sunday *et al.*, (2014b), Sunday *et al.*, (2015a), Sunday *et al.*, (2015b), Sunday *et al.*, (2015c), Sunday *et al.*, (2015d), among others.

An Overview of Electrical Circuits

Electrical circuits are described by differential equations for time-dependent elements (capacitors, inductances) together with equations for linear and non-linear time-independent elements (resistors, diodes and transistors). Well-known Ohm's and Kirchhoff's laws are part of the electronic circuit description.

Equations of the form (1) are applicable to series circuits containing an electromotive force (emf), resistors, inductors and capacitors. It is important to note that the emf voltage denoted by E is measured in volt (V), current i is measured in ampere, charge q is measured in coulomb, resistance R is measured in ohm (Ω), inductance, L is measured in Henry (H) and capacitance, C is measured in Farad.

Electromotive force (for example, a battery or generator) produces a flow of current in a closed circuit and that this current produces a so called voltage drop across each resistor, inductor and capacitor, Raisinghania (2014).

We state below the three important laws concerning voltage drop across resistor, inductor and capacitor.

Law 1: The voltage drop E_R across a resistor is given by

$$E_R = Ri \tag{2}$$

where R is a constant of proportionality called resistance and i the current.

Law 2: The voltage drop E_L across an inductor is given by,

$$E_L = L\left(\frac{di}{dt}\right) \tag{3}$$

where L is a constant of proportionality called inductance.

Law 3: The voltage drop E_C across a capacitor is given by,

$$E_C = \frac{q}{C} \tag{4}$$

where C is a constant of proportionality called capacitance and q is instantaneous charge on the capacitor.
The fundamental law in the study of electric circuits is the following.

Law 4 (The Kirchhoff's Voltage Law): The sum of the voltage drops across resistors, inductors and capacitors is equal to the total emf in a closed circuit.
Thus, the relationship between Law 4 and Laws 1, 2 and 3 is given by,

$$L\left(\frac{di}{dt}\right)+Ri+\frac{q}{C}=E \tag{5}$$

containing two dependent variables i and q. But, we also have,

$$i=\frac{dq}{dt}\text{, so that }\frac{di}{dt}=\frac{d^2q}{dt^2} \tag{6}$$

Using (6), (5) takes the form,

$$L\left(\frac{d^2q}{dt^2}\right)+R\left(\frac{dq}{dt}\right)+\frac{q}{C}=E \tag{7}$$

which is a second-order linear differential equation in the single independent variable q. It is important to note that equation (7) is of the form (1).

Derivation of the One-Step Computational Method

We shall derive the one-step computational method (using the Scientific Workplace 5.5 software) for the determination of charge on capacitors using the Hermite polynomial basis function of the form

$$y(x)=\sum_{n=0}^{6}(-1)^n e^{x^2}\frac{d^n}{dx^n}e^{-x^2}=-109+110x+676x^2-152x^3-464x^4+35x^5+64x^6 \tag{8}$$

Interpolating (8) at $x_{n+s}, s=\frac{1}{4},\frac{1}{2}$ and collocating its second derivative at $x_{n+r}, r=0\left(\frac{1}{4}\right)1$ (s *and* r are the numbers of interpolation and collocation points respectively) gives a system of non linear equation of the form,

$$XA=U \tag{9}$$

where

$$A=\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \end{bmatrix}^T, U=\begin{bmatrix} y_{n+\frac{1}{4}} & y_{n+\frac{1}{2}} & f_n & f_{n+\frac{1}{4}} & f_{n+\frac{1}{2}} & f_{n+\frac{3}{4}} & f_{n+1} \end{bmatrix}^T$$

and

$$X=\begin{bmatrix} -109 & 110x_{n+\frac{1}{4}} & 676x^2_{n+\frac{1}{4}} & -152x^3_{n+\frac{1}{4}} & -464x^4_{n+\frac{1}{4}} & 35x^5_{n+\frac{1}{4}} & 64x^6_{n+\frac{1}{4}} \\ -109 & 110x_{n+\frac{1}{2}} & 676x^2_{n+\frac{1}{2}} & -152x^3_{n+\frac{1}{2}} & -464x^4_{n+\frac{1}{2}} & 35x^5_{n+\frac{1}{2}} & 64x^6_{n+\frac{1}{2}} \\ 0 & 0 & 1352 & -912x_n & -5568x_n^2 & 700x_n^3 & 1920x_n^4 \\ 0 & 0 & 1352 & -912x_{n+\frac{1}{4}} & -5568x^2_{n+\frac{1}{4}} & 700x^3_{n+\frac{1}{4}} & 1920x^4_{n+\frac{1}{4}} \\ 0 & 0 & 1352 & -912x_{n+\frac{1}{2}} & -5568x^2_{n+\frac{1}{2}} & 700x^3_{n+\frac{1}{2}} & 1920x^4_{n+\frac{1}{2}} \\ 0 & 0 & 1352 & -912x_{n+\frac{3}{4}} & -5568x^2_{n+\frac{3}{4}} & 700x^3_{n+\frac{3}{4}} & 1920x^4_{n+\frac{3}{4}} \\ 0 & 0 & 1352 & -912x_{n+1} & -5568x^2_{n+1} & 700x^3_{n+1} & 1920x^4_{n+1} \end{bmatrix}$$

Solving (9), for $a_j\text{'s}, j = 0(1)6$ using Gaussian elimination method and substituting into (8) gives a continuous hybrid linear multistep method of the form,

$$y(x) = \alpha_{\frac{1}{4}} y_{n+\frac{1}{4}} + \alpha_{\frac{1}{2}} y_{n+\frac{1}{2}} + h^2 \left(\sum_{j=0}^{1} \beta_j(x) f_{n+j} + \beta_k(x) f_{n+k} \right),\ k = \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \tag{10}$$

The coefficients of $y_{n+j},\ j = \frac{1}{4}, \frac{1}{2}$ *and* $f_{n+j},\ j = 0\left(\frac{1}{4}\right)1$ give,

$$\left.\begin{aligned}
\alpha_{\frac{1}{4}} &= 2 - 4t \\
\alpha_{\frac{1}{2}} &= 4t - 1 \\
\beta_0 &= \frac{1}{11520}\left(4096t^6 - 15360t^5 + 22400t^4 - 16000t^3 + 5760t^2 - 962t + 57\right) \\
\beta_{\frac{1}{4}} &= -\frac{1}{2880}\left(4096t^6 - 13824t^5 + 16640t^4 - 7680t^3 + 882t - 153\right) \\
\beta_{\frac{1}{2}} &= \frac{1}{1920}\left(4096t^6 - 12288t^5 + 12160t^4 - 3840t^3 + 66t + 7\right) \\
\beta_{\frac{3}{4}} &= -\frac{1}{2880}\left(4096t^6 - 10752t^5 + 8960t^4 - 2560t^3 + 70t - 3\right) \\
\beta_1 &= \frac{1}{11520}\left(4096t^6 - 9216t^5 + 7040t^4 - 1920t^3 + 54t - 3\right)
\end{aligned}\right\} \tag{11}$$

where $t = (x - x_n)/h$, $y_{n+j} = y(x_n + jh)$ and

$f_{n+j} = f\left((x_n + jh), y(x_n + jh), y'(x_n + jh)\right)$.

Solving (10) for the independent solution at the grid points gives the continuous block method,

$$y(x) = \sum_{j=0}^{1} \frac{(jh)^{(m)}}{m!} y_n^{(m)} + h^2 \left(\sum_{j=0}^{1} \sigma_j(x) f_{n+j} + \sigma_k f_{n+k} \right),\ k = \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \tag{12}$$

The coefficients of f_{n+j} *and* f_{n+k} give,

$$\left.\begin{aligned}
\sigma_0 &= \frac{1}{90}\left(192t^5 - 600t^4 + 700t^3 - 375t^2 + 90t\right)\\
\sigma_{\frac{1}{4}} &= -\frac{1}{45}\left(384t^5 - 1080t^4 + 1040t^3 - 360t^2\right)\\
\sigma_{\frac{1}{2}} &= \frac{1}{15}\left(192t^5 - 480t^4 + 380t^3 - 90t^2\right)\\
\sigma_{\frac{3}{4}} &= -\frac{1}{45}\left(384t^5 - 840t^4 + 560t^3 - 120t^2\right)\\
\sigma_1 &= \frac{1}{90}\left(192t^5 - 360t^4 + 220t^3 - 45t^2\right)
\end{aligned}\right\} \qquad (13)$$

Evaluating (12) at $t = \frac{1}{4}\left(\frac{1}{4}\right)1$ gives a discrete one-step computational block method as,

$$A^{(0)}Y_m^{(i)} = \sum_{i=0}^{1} h^i e_i y_n^{(i)} + h^2 d_i f\left(y_n\right) + h^2 b_i f\left(Y_m\right),\ i = 0,1 \qquad (14)$$

where

$$\mathbf{Y}_m = \left[\, y_{n+\frac{1}{4}}\ \ y_{n+\frac{1}{2}}\ \ y_{n+\frac{3}{4}}\ \ y_{n+1} \right]^T,\ f(\mathbf{Y}_m) = \left[\, f_{n+\frac{1}{4}}\ \ f_{n+\frac{1}{2}}\ \ f_{n+\frac{3}{4}}\ \ f_{n+1} \right]^T$$

$$\mathbf{y}_n^{(i)} = \left[\, y_{n-1}^{(i)}\ \ y_{n-2}^{(i)}\ \ y_{n-3}^{(i)}\ \ y_n^{(i)} \right]^T,\ f(\mathbf{y}_n) = \left[\, f_{n-1}\ \ f_{n-2}\ \ f_{n-3}\ \ f_n \right]^T$$

and $A^{(0)} = 4\times 4$ identity matrix.

When $i = 0$:

$$e_0 = \begin{bmatrix} 0&0&0&1\\ 0&0&0&1\\ 0&0&0&1\\ 0&0&0&1 \end{bmatrix},\ e_1 = \begin{bmatrix} 0&0&0&\frac{1}{4}\\ 0&0&0&\frac{1}{2}\\ 0&0&0&\frac{3}{4}\\ 0&0&0&1 \end{bmatrix},\ d_0 = \begin{bmatrix} 0&0&0&\frac{367}{23040}\\ 0&0&0&\frac{53}{1440}\\ 0&0&0&\frac{147}{2560}\\ 0&0&0&\frac{7}{90} \end{bmatrix},\ b_0 = \begin{bmatrix} \frac{3}{128}&\frac{-147}{3840}&\frac{29}{3760}&\frac{-7}{7680}\\ \frac{1}{10}&\frac{-1}{48}&\frac{1}{90}&\frac{-1}{480}\\ \frac{117}{640}&\frac{27}{1280}&\frac{3}{128}&\frac{-9}{2560}\\ \frac{4}{15}&\frac{1}{15}&\frac{4}{45}&0 \end{bmatrix}$$

When $i = 1$:

$$e_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; d_1 = \begin{bmatrix} 0 & 0 & 0 & \frac{251}{2880} \\ 0 & 0 & 0 & \frac{29}{360} \\ 0 & 0 & 0 & \frac{27}{320} \\ 0 & 0 & 0 & \frac{7}{90} \end{bmatrix}, \; b_1 = \begin{bmatrix} \frac{323}{1440} & \frac{-11}{120} & \frac{53}{1440} & \frac{-19}{2880} \\ \frac{31}{90} & \frac{1}{15} & \frac{1}{90} & \frac{-1}{360} \\ \frac{51}{160} & \frac{9}{40} & \frac{21}{160} & \frac{-3}{320} \\ \frac{16}{45} & \frac{2}{15} & \frac{16}{45} & \frac{7}{90} \end{bmatrix}$$

Analysis of Basic Properties of the Computational Method

Order of the One-Step Computational Method

Let the linear operator $L\{y(x);h\}$ associated with the discrete computational block method (14) be defined as,

$$L\{y(x);h\} = A^{(0)}\mathbf{Y}_m^{(i)} - \sum_{i=0}^{1} h^i e_i y_n^{(i)} - h^2\left(d_0 f(y_n) + b_0 F(\mathbf{Y}_m)\right) \qquad (15)$$

Expanding (15) in Taylor series and comparing the coefficients of h gives,

$$L\{y(x);h\} = \bar{c}_0 y(x) + \bar{c}_1 h y'(x) + \bar{c}_2 h^2 y''(x) + \ldots + \bar{c}_p h^p y^p(x) + \bar{c}_{p+1} h^{p+1} y^{p+1}(x) + \bar{c}_p h^{p+2} y^{p+2}(x) \ldots \qquad (16)$$

Definition 1 (Lambert, 1973): *The linear operator* L *and the associated block formula (14) are said to be of order* p *if* $\bar{c}_0 = \bar{c}_1 = \bar{c}_2 = \ldots = \bar{c}_p = \bar{c}_{p+1} = 0$ *and* $\bar{c}_{p+2} \neq 0.$

c_{p+2} is called the error constant and implies that the local truncation error is given by,

$$t_{n+k} = \bar{c}_{p+2} h^{(p+2)} y^{(p+2)}(x) + O(h^{p+3}) \qquad (17)$$

Expanding the newly derived computational method in Taylor series and comparing the Coefficients of h gives $\bar{c}_0 = \bar{c}_1 = \bar{c}_2 = \bar{c}_3 = \bar{c}_4 = \bar{c}_5 = \bar{c}_6 = 0$ and the error constant is given by

$$\bar{c}_7 = \left[6.4790\times10^{-7} \;\; 1.5501\times10^{-6} \;\; 2.4523\times10^{-6} \;\; 3.1002\times10^{-6}\right]^T$$

Therefore, the one-step computational method is of uniform fifth order.

Zero Stability of the One-Step Computational Method

Definition 2 (Fatunla, 1988): *The block method (14) is said to be zero-stable, if the roots* $z_s, s = 1, 2, \ldots, k$ *of the first characteristic polynomial* $\rho(z)$ *defined by* $\rho(z) = \det(zA^{(0)} - e_0)$ *satisfies* $|z_s| \le 1$ *and every root satisfying* $|z_s| = 1$ *have*

multiplicity not exceeding the order of the differential equation. Moreover, as $h \to 0,\ \rho(z) = z^{r-\mu}(z-1)^{\mu}$ *where* μ *is the order of the differential equation,* r *is the order of the matrices* $A^{(0)}$ *and* e_0, see Awoyemi (1999) for details.
For our computational method,

$$\rho(z) = \left| z \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right| = 0 \tag{18}$$

$\rho(z) = z^3(z-1) = 0, \Rightarrow z_1 = z_2 = z_3 = 0, z_4 = 1$. Hence, the computational method is zero-stable.

Consistency of the One-Step Computational Method
The computational block method (14) is consistent since it has order $p = 5 \geq 1$.

Convergence of the One-Step Computational Method
The computational method is convergent by consequence of Dahlquist theorem stated below.

Theorem 1 *(Dahlquist, 1956)**:** The necessary and sufficient conditions that a continuous LMM be convergent are that it be consistent and zero-stable.*

Region of Absolute Stability of the One-Step Computational Method
Definition 3 (Yan, 2011)***:*** *Region of absolute stability is a region in the complex* z *plane, where* $z = \lambda h$. *It is defined as those values of* z *such that the numerical solutions of* $y'' = -\lambda y$ *satisfy* $y_j \to 0$ *as* $j \to \infty$ *for any initial condition.*
We shall adopt the boundary locus method to determine the region of absolute stability of the computational method. This gives the stability polynomial,

$$\begin{aligned} \bar{h}(w) = & -h^8\left(\frac{7}{3686400}w^3 - \frac{521}{1848115200}w^4\right) - h^6\left(\frac{7019}{554434560}w^4 + \frac{1272109}{249495520}w^3\right) \\ & - h^4\left(\frac{5309}{34652160}w^4 + \frac{349109}{10395648}w^3\right) - h^2\left(\frac{5}{12}w^4 + \frac{59}{96}w^3\right) - w^4 - 2w^3 \end{aligned} \tag{19}$$

This gives the stability region shown in the figure below.

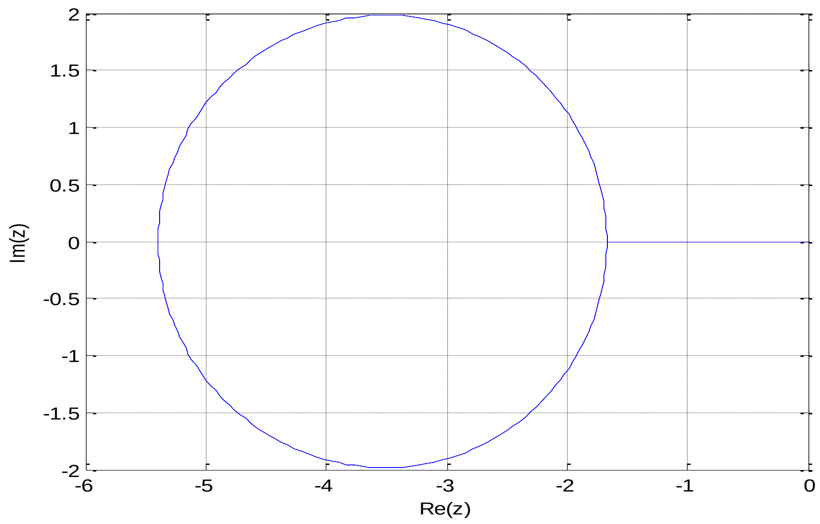

Figure 1: Absolute Stability Region of the One-Step Computational Method

By virtue of the figure obtained above, the stability region is A-stable; see Lambert (1973) for details.

Numerical Implementation and Results

We shall test the performance of the one-step fifth-order computational method developed on two problems in electricity that has to do with the determination of charge on capacitor. The following notations shall be used in the tables below;

ERR - |Exact Solution - Computed Solution|

t - Time

q - Charge on capacitors

$EvlTime$ - Evaluation time per seconds

The results for the problems to be considered below were programmed using MATLAB software version R2010a.

Numerical Experiments

Problem 5.1

An RCL circuit connected in series has resistance $R = 180\Omega$, capacitance $C = 1/280\,Farads$, inductance $L = 20\,H$ and an applied voltage $E = 10\sin t\,V$. Assuming no initial charge q on the capacitor, but an initial current i of $1\,ampere$ at time $t = 0$ when the voltage is first applied. Compute the subsequent charge on the capacitor for $t: 0.10 \le t \le 1.00$.

Source: Bronson and Costa (2006)

The first thing we do is to model this electrical circuit problem into a mathematical equation in the form of differential equation of the form (1) using (7) and then apply our method to compute the charge on the capacitor.
Thus, problem 5.1 boils down to;

$$\frac{d^2q}{dt^2}+9\frac{dq}{dt}+14q=\frac{1}{2}\sin t,\ q(0)=0,\ q'(0)=1 \tag{20}$$

The exact solution of (20) is given by,

$$q(t)=\frac{1}{500}\left(110e^{-2t}-101e^{-7t}+13\sin t-9\cos t\right) \tag{21}$$

Problem 5.2

A circuit has in series an emf given by $E=100\sin 60tV$, a resistor of 2Ω, an inductor of $0.1H$ and a capacitor of $1/260\,Farads$. If the initial current and the initial charge on the capacitor are both zero, find the charge on the capacitor at time $t:0.01\le t\le 0.10$ using a computational method.
Source: Raisinghania (2014)
The initial value problem modeling this problem is given by,

$$\left(\frac{1}{10}\right)\frac{d^2q}{dt^2}+2\frac{dq}{dt}+260q=100\sin 60t,\ q(0)=q'(0)=0 \tag{22}$$

The exact solution of (22) is given by,

$$q(t)=0.77e^{-10t}\cos(50t-0.88)-0.64\cos(60t-0.69) \tag{23}$$

Table 1: Result for Problem 5.1 for Charge in Coulomb against Time in Seconds

t	Exact Solution (q)	Computed Solution (q)	ERR	$EvlTime$
0.1000	0.0644961281691044	0.0644960195589181	1.086102e-007	0.0237
0.2000	0.0851820276111654	0.0851832385433017	1.210932e-006	0.0246
0.3000	0.0864898300025174	0.0864921392030200	2.309201e-006	0.0250
0.4000	0.0801145184634677	0.0801173973115906	2.878848e-006	0.0255
0.5000	0.0715021834960916	0.0715051970208783	3.013525e-006	0.0259
0.6000	0.0630582833428447	0.0630611589471033	2.875604e-006	0.0264
0.7000	0.0557296227838859	0.0557322207776341	2.597994e-006	0.0269
0.8000	0.0497808030832333	0.0497830714246147	2.268341e-006	0.0273
0.9000	0.0451723420652318	0.0451742794578643	1.937393e-006	0.0278
1.0000	0.0417423662543916	0.0417439972971136	1.631043e-006	0.0283

Table 2: Result for Problem 5.2 for Charge in Coulomb against Time in Seconds

t	Exact Solution (q)	Computed Solution (q)	ERR	$EvlTime$
0.0100	0.0096139444011168	0.0092268210692248	3.871233e-004	0.0610
0.0200	0.0673325959198382	0.0639508054908489	3.381790e-003	0.0897
0.0300	0.1796773416745667	0.1745050920113608	5.172250e-003	0.1182
0.0400	0.3136788440022684	0.3083558886639525	5.322955e-003	0.1464
0.0500	0.4081975148968703	0.4042992545051606	3.898260e-003	0.1673
0.0600	0.4023205217714867	0.4008943839633463	1.426138e-003	0.1681
0.0700	0.2655317378057003	0.2668092880067653	1.277550e-003	0.1686
0.0800	0.0167293959285759	0.0200983244758342	3.368929e-003	0.1690
0.0900	-0.2763823644829858	-0.2721442460399023	4.238118e-003	0.1695
0.1000	-0.5182689341485450	-0.5145781339107192	3.690800e-003	0.1700

Discussion of Results

From the results obtained in Tables 1 and 2 above, it is clear that the computational method derived is convergent because the computed solutions agree with the exact solutions. Thus, at a particular time t, one is able to know the charge q that is on the capacitor in the circuit. The evaluation time per seconds ($EvlTime$) in Tables 1 and 2 are also seen to be very small; implying that the computational method generates results very fast. Therefore, this method has a greater advantage over manual computations where one has to spend hours before computing the results.

Conclusion

We developed a one-step fifth-order computational method for determining the charge on capacitors in closed circuits. From the results obtained, it is obvious that the method is computationally reliable. The method has also been shown to be convergent, consistent and stable. Furthermore, the stability region of the method shows that it is A-stable; implying that it can efficiently cope with oscillatory and stiff problems. Finally, it is important to state that this method does not only compute charge on capacitors but can efficiently solve any real-life problem that can be modeled into second order differential equation of the form (1) be they linear or non-linear.

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