Adamawa State University Journal of Scientific Research
Volume 9 Issue 2, 2021; Article no. ADSUJSR 0902001
ISSN: 2705-1900 (Online); ISSN: 2251-0702 (Print)
http://www.adsujsr.com

Combinatorial Magic Right-Angle Triangle Characterization on Partial $t^{\Delta}-$ Symmetric Contraction Semigroups

Akinwunmi S.A.[1], Mogbonju M.M.[2], Adeniji A.O.[2], Ibrahim G.R.[3], and Fatai, M.O.[4]
[1]Department of Mathematics and Computer Science, Faculty of Science, Federal University of Kashere, P.M.B. 0182, Gombe
[2]Department of Mathematics, Faculty of Physical Science, University of Abuja, P.M.B. 117, Abuja
[3]Department of Statistics and Mathematical Science, Faculty of Science, Kwara State University Malete, P.M.B. 1530, Kwara
[4]Department of Mathematics, Faculty of Sciences, Federal University Oye-Ekiti P. M. B. 373, Ekiti
Contact: sakinwunmi@fukashere.edu.ng

(Received in June 2021; Accepted in August 2021)

Abstract
Let $M_n = \{m_1, m_2, \dots m_n\}$ be n-element distinct non-negative integer, let P_n, T_n, I_n, CI_n, ECI_n, MIC_n be partial transformation semigroup, full transformation semigroup, symmetric inverse semigroup, contraction (one-one) symmetric inverse, contraction idempotent, magic right-angle triangle contraction symmetric inverse respectively. The semigroup $(S_n,*)$ of any given partial contraction one-one transformation $\alpha \in S_n: D(\alpha) \subseteq M_n \longrightarrow I(\alpha)$ is said to be t^{Δ}-symmetric if $D(\alpha) \subseteq M_n: c_n \subseteq b_n$ where $|c_n| \leq |b_n|$ such that $(S_n,*)$ is closed under basic counting principle (sums), contains a constant (identity) element and generate magic right-angle triangles using some combinatorial parameters. This paper investigates some combinatorial parameters $(r(\alpha), b(\alpha), k^{+}(\alpha)$, and $k^{-}(\alpha))$ to characterize magic right-angle triangle for all $m, n \in M_n$, $|\alpha m - \alpha n| \leq |m - n|$ is contraction mapping such that $\alpha m, \alpha n \in D(\alpha)$, provided that any element in $D(\alpha)$ is not assumed to be mapped to empty $\emptyset$ as contraction. For a given $\alpha \in S_n$ there exist $t \in S: \{t = |Max(n, w^{+})| * |Min(n, w^{-})|\}$ for all $n \geq 1; n \in N$ then $(n; k^{+}(\alpha), k^{-}(\alpha)) = \sum_{n=1}^{k} \binom{2^{k-1}}{k+n-1}$ such that ECI_n is $t^{\Delta} -$ symmetric, also if $|E(S)| = \frac{n(n+1)}{2} + 1$, then $f(n; p, m) = \sum_{m=2}^{n} \binom{n}{m} \frac{n(n-1)}{m}$ for all $n \geq 2; n, m \in M_n$.

Keywords: contraction, magic sum, right-angle-triangle, combinatorial, semigroup
Mathematics Subject Classification: 20M20, 20M20, 20M20

Introduction

It is a reality that the implementation use of number array in the magic right-angle triangle can be a tool to develop algebraic reasoning, since the most important aspect of mathematical system is reasoning. A mathematical (algebraic) system is a set of elements together with one or more binary operations defined on the set. An algebraic system consisting of a non-empty set together with an associative binary operation $(*)$ is said to be a semigroup $(S,*)$ if it satisfies the following axioms:

i. $m, n \in S \Longrightarrow m * n \in S$
ii. $m, n, q \in S \Longrightarrow m * (n * q) = (m * n) * q$

Semigroup theory (sets which are closed under a single-valued binary operation) is one of the most natural generalization of group theory (concept) which has been in existence in algebra for many decades. In fact, the very term "Semigroup" was formed from the well-established term "group", meaning *half a group*. (Adeniji, 2013), (Howie, 1995), and (Umar, 2014) remarked that the most natural occurring semigroups of this generalization is the transformation semigroup.

Let M be a non-empty set of distinct elements and let T be set of all mappings of M *into* M, then T is a semigroup with identity under the operation $(*)$ of composition such that the above axioms is satisfied together with the following axioms:

iii. $m * m^{-1} * m = (m^{-1})^{-1} = m$

iv. $(m * n)^{-1} = n^{-1} * m^{-1}$

v. $mm^{-1} * nn^{-1} = nn^{-1} * mm^{-1}$ for all $m, n, q \in S$

then the semigroup T is called the $t^{\triangle}$-symmetric inverse semigroup on M. Since two finite combinatorial properties (w^{+}, w^{-}) with same number of n of distinct sequences (elements) are isomorphic and then form magic right-angle triangle of nth order in symmetric inverse semigroups. It suffices to consider the set $M_n = \{1,2,3, \dots n\}$, we let P_n, T_n and I_n be partial transformation semigroup, full transformation semigroup and symmetric inverse semigroup of this set. For a given partial transformation $\alpha: D(\alpha) \subseteq M_n \longrightarrow I(\alpha)$ is said to be strictly partial if $D(\alpha) \subseteq M_n$ then it is called a total mapping if $D(\alpha) = M_n$ such that $D(\alpha)$ is the domain and $I(\alpha)$ image set of α. We denote its set of idempotent elements by $U(\alpha) = \{n \in D(\alpha): \alpha^n = \alpha\}$, set of fixed points by $V(\alpha) = \{q \in D(\alpha): q\alpha = q\}$. The rank of α is denoted by $r(\alpha) = |I(\alpha)|$, width of α is denoted by $b(\alpha) = |D(\alpha)|$, right-waist of α is denoted by $k^{+}(\alpha) = |Max(n, w^{+})|$ and left-waist is denoted by $k^{-}(\alpha) = |Min(n, w^{-})|$. Throughout, we let c_n be the cardinality of CI_n and some of its sub-semigroups of ECI_n, b_n denote the number of all partial contraction one-one mapping on M_n. We adjoin an identity to S by first choosing an element $e \notin S$ then defining a multiplication on $S \cup \{e\}$ to be multiplication on S with additional products $eq = qe = q$ $(q \in S \cup \{e\})$. If S has no constant element (identity) in the rectangular array of its sequence we write $S^{e} = S \cup \{e\}$ and $S^{e} = S$ if otherwise. An element $t \in S$ is called a constant (zero) of S if $tq = qt = t$ for every $q \in S$. If S has a constant element (t), then S^{0} denote the set $S - \{0\}$ together with a partial multiplication $S^{0} * S^{0} \longrightarrow S^{0}$ defined only for pairs $(m, n) \in S^{0} * S^{0}$ where $m * n \neq 0$. Thus, we have:

$$S^{1} = \begin{cases} S, & if\ S\ has\ constant\ element \\ & S \cup \{1\}, if\ otherwise \end{cases} \quad (1)$$

$$S^{t} = \begin{cases} S, & if\ S\ has\ constant\ element; |S| > 1 \\ & S \cup \{t\}, if\ otherwise \end{cases} \quad (2)$$

For any given mapping (transformation), a contraction mapping in P_n was defined by (Garba, 1990) such that: for all αm, $\alpha n \in D(\alpha)$, $|\alpha m - \alpha n| \leq |m - n|$. Then we defined $\alpha \in I_n$ symmetric partial one-one transformation provided that any element in $D(\alpha)$ is not assumed to be mapped to zero as a one-one contraction, such that CI_n, ECI_n, MIC_n are contraction (one-one) symmetric inverse, contraction idempotent, magic right-angle triangle contraction symmetric inverse respectively. The breadth of α is denoted by $b(\alpha) = |D(\alpha)|$. The height of α is denoted by $h(\alpha) = |I(\alpha)|$. The fix of α is given by $f(\alpha) = \{\alpha(m) = m\ for\ all\ m \in S\}$. Let $(S_n, *)$ be the semigroup of natural numbers then a given partial contraction one-one transformation $\alpha \in S_n: D(\alpha) \subseteq M_n \longrightarrow I(\alpha)$ is said to be $t^{\triangle}$-symmetric if $D(\alpha) \subseteq M_n: c_n \subseteq b_n$ where $|c_n| \leq |b_n|$ such that $(S_n, *)$ is closed under basic counting principle (sums), contains a constant (identity) element and generate magic right-angle triangles using some combinatorial parameters. We defined our magic right-angle triangle MIC_n as $t^{\triangle}$-symmetric inverse contraction mapping whose entries are non-negative sequences such that the sum of the sequence in every two sides is same and the third side form $nth\ order$ of $|c_n|$, for all $m, n, q \in N$ (N: $set\ of\ natural\ numbers$). We note that the order of the magic right-angle triangle for the above semigroups varies by combinatorial properties which in turns lead to open problem. It was noted by (Havey, 2018) that unlike magic-squares, there are different magic sums for magic right-angle triangles of the same order (also known as perimeter magic triangles).

This paper investigates some combinatorial parameters $(r(\alpha), b(\alpha), k^{+}(\alpha),$ and $k^{-}(\alpha))$ to characterize magic right-angle triangle for the semigroup CI_n and ECI_n. At the end of this introductory section, we give some basic preliminaries (results) in section two that we shall need in later section. In section three, we establish some combinatorial magic right-angle recurrence relations for $c_{n,n}$ the number of partial contraction symmetric (one-one) maps of M_n having at most one constant element. Finally, in section four we give some remark which serve as open problem for the generalization of magic right-angle triangle to closely related semigroups.

Basic Preliminaries

There is remarkable case with which we can characterize semigroup to illustrate many *algebraic laws* and such elements as constant (units), zeros, idempotent and many others by using some combinatorial parameters as exhibited in this paper, then it worth to note that the combinatorial properties of S_n partial symmetric semigroup have been studied and many corresponding results have been obtained; for instance the partial transformation P_n has order $c_n = (n+1)^n$ where (Garba, 1990) showed that number of idempotent in P_n is given by $c_n = \sum_{r=0}^{n} \binom{n}{r} (r+1)^{n-r}$. Then, (Adeniji, 2013) showed that order of $I(Conv_n \wedge Cont_n)$ is given by $c_n = \sum_{k=0}^{n} 2(k+1)\binom{n}{k} + \frac{n}{2}(n+1)+1\,;\ n \geq 2$ and correspondingly give the number of idempotent in $I(Conv_n \wedge Cont_n)$ as $c_n = \frac{n(n+1)}{2}+1$. Analogously, (Shailesh, 2018) showed that the number of sequences on each side of an equilateral triangles is given by $c_n = \frac{(n+3)}{3}$. We referred to (Higgins, 1992), (Howie, 1995), (Umar, 2014), (Ganyushkin and Mazorchuk, 2009) and (Sloane, (OEIS)) for other interesting results concerning partial symmetric (one-one) transformation semigroup.

Definition 1: Partial symmetric semigroup: The partial symmetric semigroup S_n is a semigroup in which there is no image ($I(\alpha)$) set that appears more than once such that $S_n = (n+1)!$.

Definition 2: Sub-semigroup: A non-empty subset B of a semigroup $(S,*)$ is called a sub-semigroup of S if $B * B \subseteq B$ (the subset $B \in S$ is closed under the same operation $(S,*)$ then B comprises a sub-semigroup).

Definition 3: $\boldsymbol{t^{\Delta}}$- symmetric contraction semigroup: A t^{Δ}- symmetric semigroup is a sub-semigroup of I_n on M_n such that is closed under sums, contains at most one constant element (identity) and generate a magic right-angle triangle using some combinatorial parameters.

Proposition 4: Let $MIC_n \in S_n$ such that $MIC_n \subseteq CI_n \subseteq S_n$ then MIC_n has at most one constant element.

Proof: Suppose $MIC_n \in CI_n$, then there exist one and only one constant element (identity) $t \in MIC_n \colon |c_n| \leq |b_n|$ under composition of bijection mapping. But by contraction principle, let MIC_n has two distinct constant elements $t\,, t^{\Delta} \in (S_n,*)$, then $t * t^{\Delta} = t$ for each $t, t^{\Delta} \in MIC_n$. Since $t * t^{\Delta} = t^{\Delta}$, where t^{Δ} is also a constant element, so we have $t * t^{\Delta} = t$. Thus, $t = t * t^{\Delta} = t^{\Delta}$. Consequently $t^{\Delta} = t$, that is if semigroup MIC_n has a constant element, there is precisely one unique element with the constant property.

Definition 5: Breadth: $\boldsymbol{b(\alpha)}$: This is the number of elements in the domain of α. That is $|D(\alpha)| = b(\alpha)$.

Definition 6: Rank: $\boldsymbol{r(\alpha)}$: This is the number of elements in the image sets of α. That is $r(\alpha) = |I(\alpha)|$ and it is denoted by $r(\alpha) = p$.

Definition 7: Identity Element ($\boldsymbol{S_e}$): If there exist an element $1 \in M$ of a semigroup of S (monoid) such that $1m = m1 = m$ for all $m \in S$. It means that 1 is an identity element of S. Thus, there exist $e \in S$ such that $me = em = m$ for all $e, m \in S$.

Definition 8: Idempotency degree (element): If a transformation $\alpha \in MIC_n$ of t^{Δ}-symmetric magic right-angle triangle semigroup is idempotent ($\alpha^m = \alpha$) for some $n \geq 1$. The index (of idempotency) $m(\alpha)$ of $\alpha (m \neq 0)$ is the unique m for which $\alpha^{m+1} \neq 0$. We note that all idempotent in MIC_n lie in CI_n since $CI_n \Supset MIC_n$.

Definition 9: Right waist: $\boldsymbol{r^{+}(\alpha)}$: This is the maximum element in the image sets of $(I(\alpha))$ of α. That is $r^{+}(\alpha) = \max(I(\alpha))$. It is denoted by $r^{+}(\alpha) = k$.

Definition 10: Left waist: $\boldsymbol{r^{-}(\alpha)}$: This is the minimum element in the image sets of $(I(\alpha))$ of α. That is $r^{-}(\alpha) = \min(I(\alpha))$. It is denoted by $r^{-}(\alpha) = k^{-}$.

Lemma 11: Let $\alpha \in MIC_n$ with $r(\alpha) < n$, then α is idempotent if and only if there exist non-empty sub-semigroup B of $D(\alpha) \colon \alpha^m = \alpha$.

Proof: Let $M_n = \{1,2,3, \dots n\}$ be distinct finite set, and let $\alpha \in (S_n,*)$ be t^{Δ} $-$symmetric map under composition of mapping. Suppose there does not exist $B \subseteq MIC_n$ with $r(\alpha) < n$, since $\alpha \in CI_n$ (one-one map) we have $I(\alpha) = (D(\alpha))\alpha \neq D(\alpha)$ and so $D(\alpha)^2 \supseteq D(\alpha)$. Similarly $m \in N$ for all $m \geq 2$ yield $D(\alpha)^{m-1} = D(\alpha)^m$ then $D(\alpha)^m = D(\alpha * \alpha^m) = (I(\alpha) \cap D(\alpha)^m)\alpha^{m-1}$, since M_n is finite thus $|I(\alpha) \cap D(\alpha)^m| = |D(\alpha)^m|$ so $I(\alpha) \cap D(\alpha)^m = D(\alpha)^m$, such that $|c_n| \leq |b_n|$, therefore $D(\alpha)^m \supseteq D(\alpha) \colon r(\alpha) \leq n$.

Conversely, suppose $B \subseteq MIC_n$ such that B is a sub-semigroup of CI_n under composition of mapping which satisfies the same axioms of $(S_n, *)$. But by contradiction, let there exist $B \neq m$ then $D(m) \neq D(B)$, which indicate that B cannot be idempotent. Precisely if there exist $B \in MIC_n$such that $B = m$ inside $D(\alpha)$ for which $\alpha^m = \alpha$, we have $m = m\alpha = m\alpha^2 = m\alpha^n \dots\dots\dots = m_n{\alpha_n}^n$. Obviously, α is idempotent for such m (constant).

Magic right-angle triangle characterization of t^{Δ} −symmetric partial contraction semigroups

The main interest in the present section is the characterization of magic right-angle triangle on t^{Δ} −symmetric partial contraction semigroups using some combinatorial properties. Firstly, we give partial transformation (one-one) on M_n as an array of the form:

$$\boldsymbol{\alpha_n} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3}\dots\dots\dots\dots & \boldsymbol{n} \\ \boldsymbol{q_1} & \boldsymbol{q_2} & \boldsymbol{q_3}\dots\dots\dots\dots & \boldsymbol{q_n} \end{pmatrix} \quad (3)$$

Suppose $\alpha \in CI_n$ be a transformation (mapping) containing n distinct elements, we study the number of distinct subset of α that will have exactly say j elements such that the empty mapping $\emptyset$ and the mapping $\alpha_n \in I_n$ are considered to be sub-semigroups of a finite semigroup S under the composition of mapping, then from section two we have that ${\alpha_n}^{(m)} = \alpha_n$ where $\alpha_i = \alpha(i)$, if $i \in D(\alpha)$ and $I(\alpha_n) = F({\alpha_n}^m)$ for all $m, n \in M_n$ such that

$$|\boldsymbol{c_n}| = \begin{pmatrix} \boldsymbol{n_1} & \boldsymbol{n_2} & \dots\boldsymbol{n_n} \\ \boldsymbol{\alpha_1} & \boldsymbol{\alpha_2} & \dots\boldsymbol{\alpha_n} \end{pmatrix}^{(\boldsymbol{m_n}\ )} = \begin{pmatrix} \boldsymbol{n_1} & \boldsymbol{n_2} & \dots\boldsymbol{n_n} \\ \boldsymbol{\alpha_1} & \boldsymbol{\alpha_2} & \dots\boldsymbol{\alpha_n} \end{pmatrix} \quad (4)$$

Since α is t^{Δ} −symmetric partial contraction mapping with $r(\alpha) < n$ such that there exist unique constant element whenever $|c_n| \leq |b_n|$, then by composition of mapping (4) form combinatorial magic right-angle triangle plane with three coordinates (i, j, k) of nth order in figure (1):

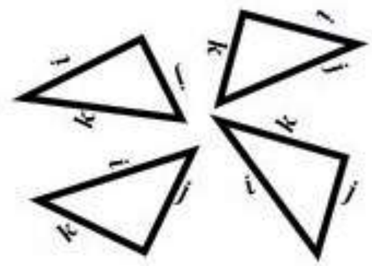
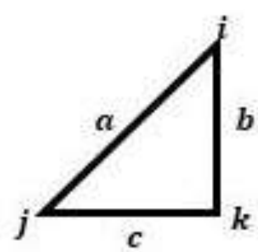


Figure 1: Three coordinate (plane)

such that the sum of the sequence in every two sides is same and the third side form $nth\ order$ for $|c_n|$ with some combinatorial parameters $(r(\alpha), b(\alpha), k^+(\alpha),$ and $k^-(\alpha))$. We discover, quite by chance some striking results as follows:

Lemma 7: The MIC_n elements of $t^{\Delta} - symmetric$ generalized sequential occurrence in rectangular array form an algebraic arithmetic progression.

Proof: For a given right-angle triangle with coordinates (i, j, k):



Let i, j, k be the elements (sequences) of the three vertices, let a, b, c the sums of the other sequences on the three edges respectively, a on edge $i - j$, b on edge $k - i$, c on edge $j - k$. Let Δ^t be the magic sum of this right-angle triangle, then we have the following relations:

$$\begin{aligned} i + a + j &= \Delta^t \\ k + b + i &= \Delta^t \qquad (5) \\ j + c + k &= \Delta^t \end{aligned}$$

By adding the three relations in (3) we get:

$2(i+j+k)+(a+b+c)=3\Delta^t$ (6)

Then we also have $i+j+k+a+b+c=1+2+3+\cdots\leq 90$ such that$i+j+k=3\Delta^t-90$. Therefore, the sum of the sequences at the vertices is $3\Delta^t-90$.

Proposition 8: Let $\alpha\in CI_n$ then the following statements hold:

i. Every element $\alpha_n\in S_n$ is a contraction idempotent
ii. $\alpha_n\in ECI_n$ has at least $k^+(\alpha)$ and $k^-(\alpha)$ for all $m,n\in D(\alpha)$
iii. There exist a unique $t\in S:\{t=|Max(n,w^+)|*|Min(n,w^-)|\}$ for all $n\geq 1; n\in N$ then $\left(n;k^+(\alpha),k^-(\alpha)\right)=\sum_{n=1}^{k}\binom{2^{k-1}}{k+n-1}$ such that ECI_n is $t^\Delta-symmetric$.

Proof: $i\Leftrightarrow ii$ Suppose $\alpha_n\in S_n$ is contraction, such that α is idempotent then $m\in M_n$ is in its range set, then there exist $n\in M_n$: $\alpha(m)n$. Since α is idempotent, $\alpha\big(\alpha(m)\big)=\alpha(m)\Longrightarrow\alpha(n)=n$ such that $k^+\in M_n$ then $|Max(n,w^+)|=|Min(n,w^-)|$ for some $i\leq 1\leq k$ then $Max(\alpha_n)$ for $m,n\in D(\alpha)$. For any given contraction mapping α_n such the $k^+(\alpha)$ and $k^-(\alpha)$ is determined by it range set we have $\alpha(m)=n: r(\alpha)<n$.

$ii\Leftrightarrow iii$ Let there exist $t\in S: t=k^+(\alpha)*k^-(\alpha)$ then under composition of inverse t^Δ −symmetric mapping by proposition (4), lemma (7) such that $D|Max(n;k^+)|$ replaced $C_{on}|Min(n;k^-)|$. Thus, we figure (2):

1					
2	2				
4	4	4			
8	8	8	8		
16	16	16	16	16	
32	32	32	32	32	32

Figure 2: Magic right-angle triangle for $f(n;k^+(\alpha),\ k^-(\alpha))$

we have a magic right-angle triangle such that the recurrence relation (value) of the magic-sum (side i,j,k) equal $\sum_{n=1}^{k}\binom{2^{k-1}}{k+n-1}$ for all $n\geq 1; n\in N$

$iii\Leftrightarrow i$ Suppose also $\alpha_n\in ECI_n$ such that $ECI_n\subseteq S_n$ where α_n is contraction then $m,n\in D(\alpha): r(\alpha_n)<M_n$where $m_{i+1}-m_i$ is the domain while $n_{i+1}-n_i$ is the image set of α_n such that $\alpha_n|m_{i+1}-m_i|\leq\alpha_n|n_{i+1}-n_i|$ for each $1\leq i\leq n-1$. Since α_n is contraction by definition then α_n is idempotent if there exist unique $t\in D(\alpha):\alpha^t*\alpha^t=\alpha$ where $\alpha^t\in S_n$ for all $t,m,n\in M_n$. The result is complete.

Theorem 9: Let $\alpha\in S$, then $|f(\alpha)|\leq|h(\alpha)|$ whenever $(n;p,(k^-))=\binom{n^2}{p-(k^-)}$ for all $n\geq 1$; $k^-\geq 0\geq p$.

Proof: let $M_n=\{1,2,3\ldots n\}$ be a finite distinct n-element set, let $\alpha\in CI_n$ such that $f(n;p,m)=|\alpha\in S: h(\alpha); f(\alpha)|=|I(\alpha)|=p(m)$ for all $m,n\in D(\alpha)$ where $|\alpha m-\alpha n|\leq|m-n|\Longrightarrow|f(\alpha)|\leq|h(\alpha)|$ such that $\alpha(m)=i$, $m\in\{i,i-1,\ldots n\}$, then under composition of contraction $t^\Delta-symmetric$ mapping we have $\sum_{i=1}^{n}(n-i+1)=1; n\geq 0$. For the second statement, since $\alpha\in CI_n$ then empty map $\emptyset$ is a subset of all mapping under composition of contraction $t^\Delta-symmetric$ mapping such that there exist at least minimum $k^-\in D(\alpha): f(n;p,k^-)=|\alpha\in S: w^-(\alpha)|\leq|I(\alpha)|$ then $\alpha(S)=(n;p,k^-)$ can be chosen from $M_n=\{1,2,3,\ldots n\}$ in $\binom{n^2}{p-(k^-)}$ ways. Thus, one can check if $w^-(\alpha)$ the $Min(n;p,k^-)$ form a magic right-angle triangle where $|Min(n;w^-)|$ switch with constant element $n(2)$ then $(n;p)$ with $n(1)$ such that the magic-sum of the triangle is given by $(n;p,k^-)=\binom{n^2}{p-(k^-)}$

Theorem 10: Let $\alpha\in S$ such that $ECI_n\subseteq CI_n\subseteq S$ where $|E(S)|=\frac{n(n+1)}{2}+1$, then $f(n;p,m)=\sum_{m=2}^{n}\binom{n}{m}\frac{n(n-1)}{m}$ for all $n\geq 2$; $n,m\in M_n$.

Proof: Suppose $\alpha \in ECI_n$ then $D(\alpha) \subseteq M_n: I(\alpha) \subseteq M_n$, since $m(\alpha)$ of domain in a set $M_n = \{1, 2, 3, \ldots n\}$ is chosen from M_n is $\binom{n}{m}$ ways then each partial bijection $\alpha: D(\alpha) \longrightarrow I(\alpha)$. If $f(n; m)$ is fixed under composition of bijection mapping where $\alpha \in ECI_n$ then $f(n; p, m) = |\alpha \in S: f(\alpha)| = I(\alpha)$, and $|\alpha \in S: m(\alpha)| = f(m)$ such that $|\alpha m - \alpha n| \leq |m - n|$ then there exist $t \in MIC_n: t = |h(\alpha)| * |m(\alpha)|$, by proposition (4), lemma (7) such that $D|F(n; p)|$ was interchanged with $C_{on}|F(n; m)|$ then we have figure (3)

1					
3	3				
6	6	6			
10	10	10	10		
15	15	15	15	15	
21	21	21	21	21	21

Figure 3: Magic right-angle triangle for $f(n; h(\alpha),\ m(\alpha))$

such that the recurrence relation (value) of the magic-sum (side i, j, k) is given by $\sum_{m=2}^{n} \binom{n}{m} \frac{n(n-1)}{m}$ for all $n \geq 2$; $n, m \in M_n$:

Concluding Remarks

Remark 1: For any given magic right-angle triangle of order, we have $\frac{2}{3}(n-1)$ distinct positive integers placed in a triangular array, n integers in each side, so that the sum of the integers on each side is a magic sum.

Remark 2: In ECI_n, two finite combinatorial properties say $[k^{+}(\alpha)$ and $k^{-}(\alpha)$] with same number of distinct sequences are isomorphic and form a magic right-angle triangle of nth order. It is obvious to observe that the order of number of sequences on each side of a non-magic right-angle triangle is given by $|c_n| = \frac{n+3}{3}$.

Remark 3: The semigroup MIC_n can be used to discover and illustrate many algebraic laws and application which create open problem such that the generalization of the magic right-angle triangle is obtained.

Remark 4: The triangular array (sequence) of MIC_n, ECI_n and CI_n are not yet listed in (Sloane, (OEIS)), but for more useful results concerning magic triangle we refer to (Shailesh, 2018).

References:

Adeniji, A. O., (2013). On the study of cardinality and idempotency of partial one-one convex and contraction transformation semigroups. *African Journal of Math. and Comp. Sci. Research*. pp. 118-120.

Ganyushkin, O. and Mazorchuk, V., (2009). Classical finite transformation semigroups; an introduction. *Springer-Verlag London Limited.*

Garba, G.U., (1990). Idempotents in Partial transformation semigroups. *Proc. Roy. Soc. Edingburgh*. 116A: pp. 359-366.

Havey, H. (2018). Perimeter magic triangles. http://www.magicsquares.net/perimeter.htm; #order 4 perimeter magic triangles.

Higgins, P. N., (1992). Techniques of semigroup theory. *Oxford University press.*

Howie, J. M., (1995). Fundamentals of semigroup theory. *Oxford University press.*

Shailesh, A. S., (2018). Theorem concerning a magic triangle. *Azim Premji University at right angles*. pp. 86-90.

Sloane, N. J. A. On-line Encyclopedia of Integer Sequence. (OEIS). http://oeis.org//

Umar, A., (2014). Some combinatorial problems in the theory of partial transformation semigroups. *Algebra and Discrete Math*. Vol. 17, No 1, pp. 110-134