



## Determination of Thermal Diffusivity of Soil as influenced by Moisture using Analytical Methods

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### Abstract

Soil thermal diffusivity for different soil layer was estimated using time series soil surface and subsurface temperature data at the metrological station of Adamawa State University, Mubi. Hourly temperature measurement for short period (one day) was used under a wet and dry soil condition in the month of August and November respectively. Four analytical methods viz: amplitude, phase, logarithm and arctangent methods were used in this work. The values of the thermal diffusivity obtained from the different methods at deeper depth for a dry soil condition, seems to be much smaller as compared with what is observed in the wet soil condition. This implies that soil moisture content affect its thermal diffusivity.

**Keywords:** Soil, Thermal diffusivity, Temperature, Time series,

### Introduction

Soil is a dispersed polyphase system consisting of solid, liquid and gaseous phase. Each phase is physically or chemically different and mechanically separable (Rajan and Rao, 2005). Soil as a dispersed polyphase system has a thermal properties: - heat capacity, heat flux, thermal diffusivity and thermal conductivity which are important in any agricultural, engineering and meteorological applications. These thermal properties influence the partitioning of energy at the ground surface and are related to temperature and transfer of heat and water across the ground surface (Ochsner, Horton and Ren, 2001). Soil temperature is a major influence of the soil thermal properties. It is the measure of the temperature at a depth below the earth surface. The ground surface get heated more during the day by intense solar radiation than the layer beneath, resulting in temperature gradient between the surface and subsoil on one hand and surface and air layer near the ground on the other. Within the soil this causes heat flow as a thermal wave, the amplitude of which changes with depth. When the incoming solar radiation falls on the soil surface, it heats the subsoil thus causing a different between the surface and the subsoil.

These differences causes the heat to flow downward in the form of wave refers to as the temperature wave (Ghilyal and Tripathi, 2005).

Thermal diffusivity is a composite parameter that indicates how fast a temperature change occurs in a material subjected to the thermal gradient. It is the change in temperature produced in a unit volume by a quantity of heat flowing through the temperature gradient. It can be measured from temperature data determined at various depth and different time of the day.

Thermal parameters of a soil depend on several factors, such as soil texture, mineralogical composition, the presence of salt, soil moisture content (Wang et al., 2012) and therefore making estimations of thermal parameters difficult. The objectives of this work is to determine the influence of moisture content on thermal diffusivity of soil using four different analytical method on a dry and wet soil condition.

### Materials and Methods

Methods have been developed to estimate thermal diffusivity  $K$  using field data

on soil temperature (Horton, Wierenga and Nielson, 1983). Soil temperature measured at shallow depth generally show diurnal cyclic variations, which can be described by the one-dimensional heat conduction equation. The thermal diffusivity  $K$  is the key parameter that controls the temperature in soils near the

ground surface in a vertical temperature gradient. The rate at which heat flows through a soil at any level  $z$  below the surface is directly proportional to the vertical temperature gradient existing at that level. We relate the rate of heat conduction to the temperature gradient as:

$$\frac{dQ}{dt} = -KA \frac{dT}{dz} \quad (1)$$

Where:

$dQ$  is the amount of heat movement in Joules

$dt$  is the time interval in seconds

$dT$  is the temperature change in degree centigrade

$dz$  is the depth increment in meters

$A$  is the cross sectional area in square meters, and

$K$  ( $m^2d^{-1}$ ) is the thermal diffusivity defined as the ratio between thermal conductivity and volume-based specific heat. It is the quantity of heat flowing in unit time through the soil cross section in response to a specified temperature gradient (it varies considerably with soil type and moisture content) the negative sign indicates that heat flows from hot to cool regions in the soil against the temperature gradient.

the following assumption were used to solve eqn. (1):

(1) the soil surface (i.e.,  $z = 0$ ) is subjected to sinusoidal temperature variation;

(2) at infinite depth, the soil temperature is constant and is equal to the average soil temperature; (3) the apparent thermal diffusivity is constant throughout the soil profile and throughout the year. Hillel (1982) and Marshall and Holmes (1988) developed a solution based on the above assumption that satisfies the one-dimensional heat equation as:

Considering the surface temperature over a day or a year to vary according to a sine-wave,

$$T(z, t) = T_a + Ae^{-z/d} \sin\left(\frac{2\pi t}{\tau} - \frac{z}{d}\right) \quad (2)$$

Where:  $T(z, t)$  is soil temperature at time  $t$ (s) and depth  $z$ (m),  $T_a$  = average soil temperature °C,  $\tau$  = period either hours or days, the parameter  $d$ (m) is called the damping depth, which is a constant characterizing the decrease

in temperature amplitude with an increase in distance from the soil surface (Carslaw and Jaeger 1959, van Wijk and de Vries 1966) defined as;

$$d = \sqrt{\frac{2k}{\omega}} \quad (3)$$

Where  $k$  is the thermal diffusivity and  $\omega$  is the angular velocity of the earth rotation given as  $\omega = 2\pi/P$  where  $P$  is the time period. When the thermal diffusivity  $k$  is known, eqn. (2) can be used to predict temperature behavior in time and depth.

In this study we aimed to determine soil thermal diffusivity from soil temperature data by four calculation procedures viz; amplitude algorithm, phase algorithm, arctangent algorithm and logarithm method.

**Amplitude method**

Soil temperature measured at two different depths ( $z_1$  and  $z_2$ ) are often assumed to be approximated by a sinusoidal function when estimating  $k$ . The sinusoidal functions are given by:

$$T(z_1, t) = \bar{T}_1 + A_1 \sin(\omega t + \phi_1) \quad (4)$$

$$T(z_2, t) = \bar{T}_2 + A_2 \sin(\omega t + \phi_2) \quad (5)$$

where  $A_1, A_2, \phi_1, \phi_2, \bar{T}_1, \bar{T}_2$  are the amplitude, phase and mean soil temperature at the depth  $z_1, z_2$ .  $A_1, A_2$  is determine as described by Gao et. al. (2009) as half the difference between the daytime maximum value and the nighttime minimum value for soil depth of  $z_1, z_2$  and  $\phi_1,$

$\phi_2$  is the initial phase of soil temperature at depth  $z_1, z_2$ , obtained by using the best fit algorithm (Horton, Wierenga and Nielson,1983). Then the thermal diffusivity  $k$  is determined by solving eqn. (4) and (5) as:

$$K = \frac{\omega}{2} \left[ \frac{z_2 - z_1}{\ln(A_1/A_2)} \right]^2 \quad (6)$$

**Phase method**

This method uses the phase lag between the sine waves at the two depths. This can be achieved by determining the times at which the temperature wave reaches its maximum (or minimum) value at the two depths. If the time

interval between the measured occurrences of maximum soil temperature at the depths of  $z_1$  and  $z_2$  is  $\Delta t = t_2 - t_1$ , then from eqn. (2) the phase algorithm is (Horton, Wierenga and Nielson, 1983):

$$K = \frac{1}{2\omega} \left[ \frac{z_2 - z_1}{\Delta t} \right] \quad (7)$$

**Logarithmic method**

This method was proposed by Seemann (1979) and quoted (with modifications) by Horton, Wierenga and Nielson, (1983). It is based on

four temperature observations during a 24h period (6h between observations), at two depths. Using the same assumption of the Arctangent algorithm,  $k$  is expressed by:

$$K = D \left[ \frac{z_2 - z_1}{\ln \left\{ \frac{[(T_1 - T_3)^2 + (T_2 - T_4)^2]}{[(T'_{13} - T'_{13})^2 + (T'_{24} - T'_{24})^2]} \right\}} \right]^2 \quad (8)$$

$D$  is a constant whose value is  $12.65 \text{ d}^{-1}$  (for  $K$  in  $\text{m}^2 \text{ d}^{-1}$ ). Horton et al. (1983) reported  $D = (0.0121)^2 \text{ s}^{-2}$  (for  $K$  in  $\text{m}^2 \text{ s}^{-1}$ )

**Arctangent method**

Soil surface temperature can be described by a Fourier series:

$$T(z, t) = \bar{T} + \sum_{i=1}^n [a_i \sin(i\omega t) + b_i \cos(i\omega t)] \quad (9)$$

where  $n$  is the number of harmonics, and  $a_i$  and  $b_i$  are the amplitudes. With boundary

condition  $n = 2$ ,  $k$  can be calculated by the Arctangent algorithm as,

$$K = \frac{\omega}{2} \left[ \frac{z_2 - z_1}{\arctan X} \right]^2 \tag{10}$$

$$\text{Where } X = \frac{(T_1 - T_3)(T'_2 - T'_4) - (T_2 - T_4)(T'_1 - T'_3)}{(T_1 - T_3)(T'_1 - T'_3) + (T_2 - T_4)(T'_2 - T'_4)} \tag{11}$$

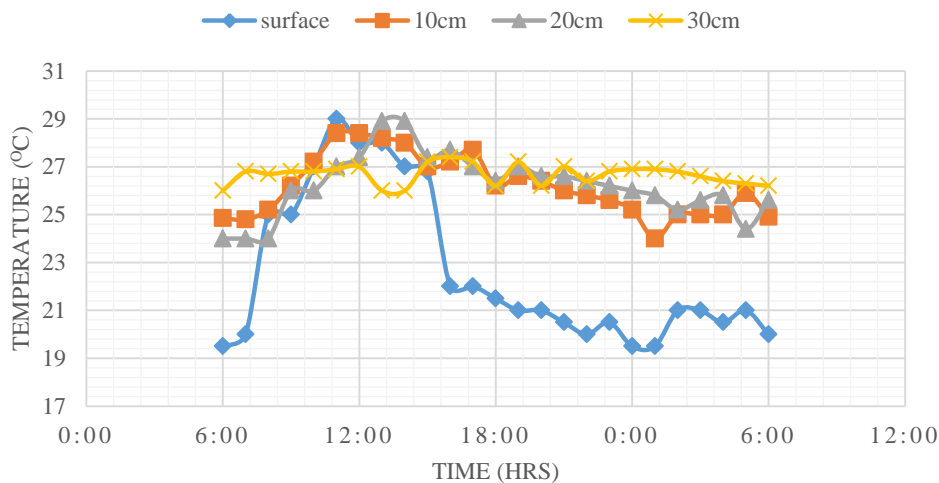
The temperatures  $T_j$  and  $T'_j$  are recorded each 6 h ( $j=1, 2, 3,$  and  $4$ ) at two different depths  $z_1$  and  $z_2$ , respectively.

**Data collection**

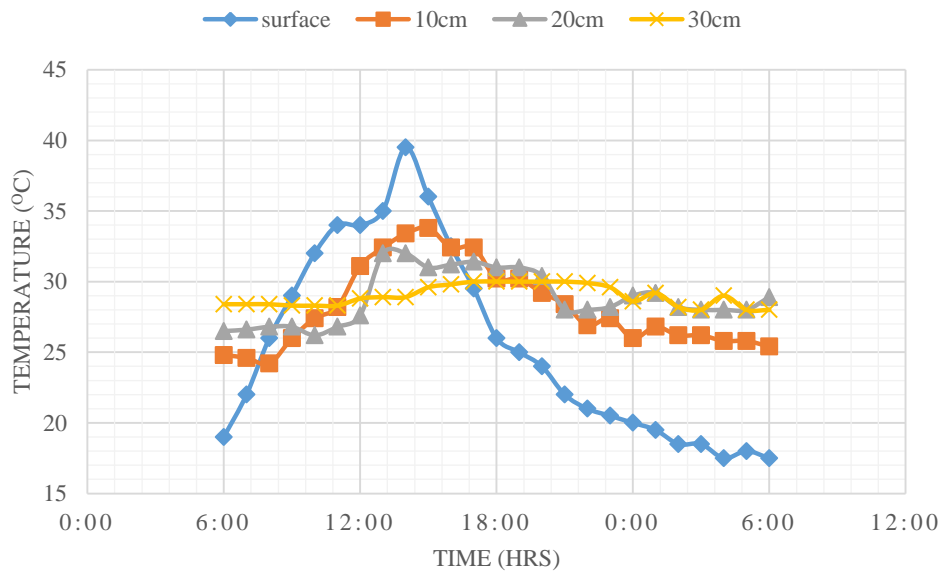
Soil temperatures were measured at two different days of the year at the meteorological station of the Geography Department, Adamawa State University, Mubi. The first data was collected on 23<sup>rd</sup> of August, 2010 while the second data on 21<sup>st</sup> November, 2010 using soil thermometer. The data were collected at four different depths (0 m, 0.1 m, 0.20 m and 0.30 m) at an observation frequency of one hour for a period of 24 hours (from 6 am to 6 am). The soil type is predominantly sandy with flat and homogeneous ground surface.

**Results and Discussion**

The temperature data collected were plotted against time at the different depths. Fig. 1 and 2 show the variation of temperature with time at the various depth for the two days. The soil temperature collected on both days was observed to change diurnally and reaches its peak at different time of the day with the time lag increasing with depth. The temperature at 30 cm remains almost constant at 30°C with little diurnal variation. The plot shows that the soil at greater depth shows less variation than those at the near surface for both soil condition which is in accordance with the findings of Rajeev and Kodikar (2015).



**Figure 1:** Temporal variation of soil temperature at different depth in August.



**Figure 2:** Temporal variation of soil temperature at different depth in November.

The information from the plot was used to estimate the thermal diffusivity for layers 0 – 10 cm, 10 – 20 cm and 20 – 30 cm using

equations (6), (7), (9) and (11). The result for the estimated values is presented in the table below.

**Table1:** Calculated thermal diffusivity values in different soil layers for different soil conditions.

<b>Under wet soil condition in August</b>				
Soil depth (cm)	Amplitude method ( $m^2 sec^{-1}$ )	Phase method ( $m^2 sec^{-1}$ )	Logarithm method ( $m^2 sec^{-1}$ )	Arctangent method ( $m^2 sec^{-1}$ )
0 – 10	$6.14 \times 10^{-6}$	$5.31 \times 10^{-6}$	$1.32 \times 10^{-6}$	$5.90 \times 10^{-7}$
10 – 20	$1.25 \times 10^{-5}$	$4.31 \times 10^{-7}$	$7.35 \times 10^{-6}$	$1.23 \times 10^{-6}$
20 – 30	$2.32 \times 10^{-6}$	$1.92 \times 10^{-7}$	$1.86 \times 10^{-7}$	$3.27 \times 10^{-7}$
<b>Under dry soil condition in November</b>				
0 – 10	$5.29 \times 10^{-6}$	$5.31 \times 10^{-6}$	$6.59 \times 10^{-7}$	$1.52 \times 10^{-7}$
10 – 20	$1.43 \times 10^{-5}$	$1.32 \times 10^{-6}$	$1.77 \times 10^{-6}$	$1.53 \times 10^{-7}$
20 – 30	$3.21 \times 10^{-6}$	$2.12 \times 10^{-7}$	$3.18 \times 10^{-6}$	$2.28 \times 10^{-7}$

The calculated values for each method show some variation for the vertical temperature gradient from the surface soil temperature with that of the other subsurface gradient. Hence using the vertical temperature from the surface to estimate the thermal diffusivity seems to overestimate the value in all the methods

except for the arctangent methods whose result seem to have no much variation under the wet soil condition.

For the dry soil condition in November, the value of thermal diffusivity at deeper depth seems to be much smaller unlike what is

observed in the wet soil condition taken in August. This is probably due to moisture content, as it is one of the factors affecting the thermal properties of soil. The variation in value may also be attributed to the inhomogeneity in composition of the soil in deeper depth (Wang *et. al.*, 2012).

### Conclusion

This work presents the application of the numerical method in determining the soil thermal diffusivity under different weather condition and the evaluation of the adequacy of the method using temperature data. The results of the four different numerical method at deeper depth for a dry soil condition seems to be much smaller as compared with what is observed in the wet soil condition. This implies that soil moisture content affect the value of soil thermal diffusivity. In terms of the adequacy of the methods, the arctangent method show a low values as compared to the other methods which agrees with the work of Gao *et. al* (2009).

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