©Adamawa State University Journal of Scientific Research 04(1): April, 2016
ISSN: 2251-0702

Danzig's Simplex Approach to the Optimal Scheduling of Premium Motor Spirit Distribution in Nigeria

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ABSTRACT
The downstream petroleum sector was examined in order to determine an effective and efficient distribution of Premium Moto Spirit in Nigeria. Three refineries and seventeen storage depots were considered in the model formulation as a linear programming problem with twenty constraints and seventeen variables and was solved using MAPLE 15 SOFTWARE which uses Danzig's simplex approach. Optimal analysis was performed investigating the supply from refineries and demand at storage depots and the optimal solution was obtained. Among other things, plant capacities of the nation's refineries should be adjusted and made more functional to meet the projected production to satisfy the demand.

KEYWORDS: Objective Function, Constraints, Optimal Solution, Slack Variable, Surplus Variable.

Introduction

Operation research started just before World War 2 in Britain with the establishment of team of scientists to study the strategic and tactical problems involved in military operations to find the most effective utilization of limited military resources by the use of quantitative techniques. Ever since, Operation Research is employed in public services for more efficient and scientific approach to the system. Techniques such as goal programming, integer programming, queuing theory are required. Hence, Operation Research is needed for proper policy making. It is needed to optimize the scheduling of premium motor spirit distribution in Nigeria.

The research work covers the three refineries and the seventeen deports. The three refineries are located at Port Harcourt, Warri and Kaduna while the depots are at Aba, Enugu, Makurdi, Yola, Benin, Ore, Mosimi, Satelite, Ibadan, Ilorin, Suleja, Minna, Jos, Gombe, Maiduguri, Kano, Gusau (NNPC 1994, PPMC 1995).

The supply and demand model incorporating both primary and secondary distributions, that is moving products from refineries or other supply sources to depots at least cost by pipelines and moving products from depots to consumers such as petrol stations by lorry tankers are represented as linear programming problem by Mehring and Gutherman (1990) in which total cost of delivery are minimized or profit contribution is maximized. An algorithm known as parallel primal-dual simplex which is capable of solving linear programs with thousands of rows and millions of columns was also developed.

Materials and Methods

The Simplex Method

The simplex method has proved to be the most effective method for solving linear programming problem as in Hu and Johnson (2000). This method is applicable to any problem that can be formulated in terms of linear objective function, subject to a set of linear constraints. In the maximization case, the method is applied to linear problems where the objective is to maximize the profit with less than or equal to the type of constraints in the form:

Maximize

$$C_1X_1 + C_2X_2 + C_3X_3 + ... + C_nX_n$$

Subject to

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n \le b_1\\
&a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + ... + a_{2n}x_n \le b_2\\
&a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + ... + a_{3n}x_n \le b_3 \qquad (1)\\
&. \qquad . \qquad .\\
&. \qquad . \qquad .\\
&. \qquad . \qquad .\\
&a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + ... + a_{mn}x_n \le b_m
\end{aligned}$$

where b_i, i=1,2,…,m are the resources, a_{ij} (i =1,2,…m; j = 1,2,…n) are the input and output coefficients. In converting to equalities, slack variables are introduced.

Hence

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n + s_1 = b_1\\
&a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + ... + a_{2n}x_n + s_2 = b_2\\
&a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + ... + a_{3n}x_n + s_3 = b_3 \qquad (2)\\
&. \qquad . \qquad .\\
&. \qquad . \qquad .\\
&. \qquad . \qquad .\\
&a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + ... + a_{mn}x_n + s_n = b_m
\end{aligned}$$

where $x_1,x_2,\ldots,x_n \ge 0$, $S_1,S_2,\ldots,S_m \ge 0$. In the minimization case, it is applied to linear programming problems where the objective function is to minimize the cost or time with greater than or equal to type of constraints in the form:

Minimize

$$C_1X_1 + C_2X_2 + C_3X_3 + ... + C_nX_n$$

Subject to

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n \ge b_1\\
&a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + ... + a_{2n}x_n \ge b_2\\
&a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + ... + a_{3n}x_n \ge b_3 \qquad (3)\\
&. \qquad . \qquad .\\
&. \qquad . \qquad .
\end{aligned}$$

. . .

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + ... + a_{mn}x_n \geq b_m$$

Next, surplus variables are introduced to give:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n - s_1 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + ... + a_{2n}x_n - s_2 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + ... + a_{3n}x_n - s_3 = b_3 \quad (4)$$

. . .
. . .
. . .

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + ... + a_{mn}x_n - s_m = b_m$$

where

$x_1, x_2, \ldots, x_n \geq 0$ and $S_1, S_2, \ldots, S_m \geq 0$

An initial basic feasible solution is obtained by setting

$x_1 = x_2 = \ldots = x_n = 0$ and $S_1 = -b_1$, $S_2 = -b_2, \ldots, S_m = -b_m$

But $S_1, S_2, \ldots, S_m$ cannot be less than zero, hence artificial variables are introduced and the set of constraints give:

Next, surplus variables are introduced to give:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n - s_1 + A_1 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + ... + a_{2n}x_n - s_2 + A_2 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + ... + a_{3n}x_n - s_3 + A_3 = b_3 \quad (5)$$

. . .
. . .
. . .

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + ... + a_{mn}x_n - s_m + A_m = b_m$$

where

$x_1, x_2, \ldots, x_n \geq 0$, $S_1, S_2, \ldots, S_m \geq 0$ and $A_1, A_2, \ldots, A_m \geq 0$

The system can be solved analytically or otherwise.

Premium Motor Spirit for the Year 2015

The projected Premium Motor Spirit (PMS) supplied by refineries and the projected PMS demanded at storage depots for the year 2015 and the unit cost of transporting one metric tonne of PMS from refineries to storage depots is given here.

The objective of the model is to minimize the cost of delivery of PMS from three refineries to seventeen storage depots through pipeline network. The optimization problem is:

$$\text{Minimize } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij} \quad (6)$$

Subject to the constraints

$$\sum_{i=1}^{m} X_{ij} \leq S_i \text{ All sources model i} \quad (7)$$

i = 1,2,3… n, n =3 in this case.

$$\sum_{j=1}^{n} X_{ij} \geq D_j \text{ All demand model j} \quad (8)$$

j=1,2…m, m=17 in this case.

$$X_{ij} \geq 0$$

Where:
X_{ij} = quantity of product transported from supply point i to demand point j
C_{ij} = cost of transporting each unit product from supply point i to demand point j
S_i = supply point availability
D_j= Demand point requirement

Table 1: Data for the Model.

Refineries Cities	Port Harcourt	Warri	Kaduna	Demand
Aba	60 x_{11}	571 x_{21}	792 x_{31}	788,863
Enugu	229 x_{12}	399 x_{22}	622 x_{32}	950,013
Makurdi	426 x_{13}	596 x_{23}	818 x_{33}	435,369
Yola	979 x_{14}	1149 x_{24}	1667 x_{25}	154,653
Benin	531 x_{15}	99 x_{25}	554 x_{35}	596,519
Ore	656 x_{16}	223 x_{26}	679 x_{36}	244,326
Mosimi	821 x_{17}	388 x_{27}	844 x_{37}	1,020,195
Satelite	868 x_{18}	445 x_{28}	890 x_{38}	1,846,744
Ibadan	907 x_{19}	474 x_{29}	930 x_{39}	1,316,504
Ilorin	1093 x_{110}	660 x_{210}	1117 x_{310}	389,883
Suleja	687 x_{111}	488 x_{211}	165 x_{311}	341,797
Minna	775 x_{112}	576 x_{212}	251 x_{312}	63,681
Jos	1031 x_{113}	833 x_{213}	180 x_{313}	230,031
Gombe	1320 x_{114}	1123 x_{214}	470 x_{314}	144,256

Maiduguri	1646 x_{115}	1447 x_{215}	794 x_{315}	171,461
Kano	1095 x_{116}	896 x_{216}	244 x_{316}	283,315
Gusau	1135 x_{117}	936 x_{217}	284 x_{317}	98,771
				9,076,381
Supply	7,201,849	3,192,156	2,603,375	12,997,380

Source: Field Survey 2015

Representation of the problem as a model

X_{ij} is the PMS produced at refinery i and sent to city j

Minimize Z=

$$60x_{11}+229x_{12}+426x_{13}+979x_{14}+531x_{15}+656x_{16}+821x_{17}+868x_{18}+907x_{19}+1093x_{110}+687x_{111}+775x_{112}+1031x_{113}+1320x_{114}+1646x_{115}+1095x_{116}+1135x_{117}+571x_{21}+399x_{22}+596x_{23}+1149x_{24}+99x_{25}+223x_{26}+388x_{27}+445x_{28}+474x_{29}+660x_{210}+488x_{211}+576x_{212}+833x_{213}+1123x_{214}+1447x_{215}+896x_{216}+936_{217}+792x_{31}+622x_{32}+818x_{33}+1667x_{34}+554x_{35}+679x_{36}+844x_{37}+890x_{38}+930x_{39}+1117x_{310}+165x_{311}+251x_{312}+180x_{313}+470x_{314}+794x_{315}+244x_{316}+284x_{317} \quad (9)$$

Subject to:

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17}+x_{18}+x_{19}+x_{110}+x_{111}+x_{112}+x_{113}+x_{114}+x_{115}+x_{116}+x_{117}\leq 7{,}201{,}849$$
$$x_{21}+x_{2}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27}+x_{28}+x_{29}+x_{210}+x_{211}+x_{212}+x_{213}+x_{214}+x_{215}+x_{216}+x_{217}\leq 3{,}192{,}156$$
$$x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}+x_{38}+x_{39}+x_{310}+x_{311}+x_{312}+x_{313}+x_{314}+x_{315}+x_{316}+x_{317}\leq 2{,}603{,}375 \quad (10)$$

$$\begin{aligned}
&x_{11}+x_{21}+x_{31}\geq 788{,}863\\
&x_{12}+x_{22}+x_{32}\geq 950{,}013\\
&x_{13}+x_{23}+x_{33}\geq 435{,}369\\
&x_{14}+x_{24}+x_{34}\geq 154{,}653\\
&x_{15}+x_{25}+x_{35}\geq 596{,}519\\
&x_{16}+x_{26}+x_{36}\geq 244{,}326\\
&x_{17}+x_{27}+x_{37}\geq 1{,}020{,}195 \qquad (11)\\
&x_{18}+x_{28}+x_{38}\geq 1{,}846{,}744\\
&x_{19}+x_{29}+x_{39}\geq 1{,}316{,}504\\
&x_{110}+x_{210}+x_{310}\geq 389{,}883\\
&x_{111}+x_{211}+x_{311}\geq 341{,}797\\
&x_{112}+x_{212}+x_{312}\geq 63{,}681\\
&x_{113}+x_{213}+x_{313}\geq 230{,}031\\
&x_{114}+x_{214}+x_{314}\geq 144{,}256\\
&x_{115}+x_{215}+x_{315}\geq 171{,}461\\
&x_{116}+x_{216}+x_{316}\geq 283{,}315\\
&x_{117}+x_{217}+x_{317}\geq 98{,}771
\end{aligned}$$

$x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{110}, x_{111}, x_{112}, x_{113}, x_{114}, x_{115}, x_{116}, x_{117}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{210}, x_{211}, x_{212}, x_{213}, x_{214}, x_{215}, x_{216}, x_{217}, x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}, x_{37}, x_{38}, x_{39}, x_{310}, x_{311}, x_{312}, x_{313}, x_{314}, x_{315}, x_{316}, x_{317}, \geq 0$ (12)

The model is formulated as a linear programming problem with twenty constraints and seventeen variables and was solved using MAPLE 15 SOFTWARE which uses Danzig's simplex approach as in the APPENDIX.

Reports

- Port Harcourt should supply 788,863 metric tonnes of PMS needed at Aba
- Port Harcourt should supply 950,013 metric tonnes of PMS needed at Enugu
- Port Harcourt should supply 435,369 metric tonnes of PMS needed at Makurdi
- Port Harcourt should supply 154,653 metric tonnes of PMS needed at Yola
- Port Harcourt should supply 375,271 metric tonnes of PMS needed at Benin
- Port Harcourt should supply 1,846,744 metric tonnes of PMS needed at Satelite town
- Warri should supply 221,248 metric tonnes of PMS needed at Benin
- Warri should supply 244,326 metric tonnes of PMS needed at Ore
- Warri should supply 1,020,195 metric tonnes of PMS needed at Mosimi
- Warri should supply 1,316,504metric tonnes of PMS needed at Ibadan
- Warri should supply 389,883 metric tonnes of PMS needed at Ilorin
- Kaduna should supply 341,797 metrictonnes of PMS needed at Suleja
- Kaduna should supply 63,681 metric tonnes of PMS needed at Minna
- Kaduna should supply 230,031 metric tonnes of PMS needed at Jos
- Kaduna should supply 144,256 metric tonnes of PMS needed at Gombe
- Kaduna should supply 171,461 metric tonnes of PMS needed at Maiduguri
- Kaduna should supply 283,315 metric tonnes of PMS needed at Kano
- Kaduna should supply 98,771 metric tonnes of PMS needed at Gusau

Conclusion

The optimal distribution schedule revealed that some routes are feasible for the distribution of PMS at minimum delivery cost while other routes are not feasible. It will be economical to supply the 788,863 metric tonnes that will be needed at Aba depot from the refinery in Port Harcourt(x_{11}) instead of pumping from refineries in Warri(x_{21}) or Kaduna(x_{31}).Similarly, the 1,846,744 metric tonnes that will be demanded at the depot in Satellite town in Lagos should be pumped from Port Harcourt refinery (x_{18}) only. Again, it is more economical to supply PMS from Port Harcourt and Warri refineries to those depots in the southern part of Nigeria compared to those depots in the northern part of Nigeria. Kaduna refinery if strengthen can supply PMS to the depots in the northern part of Nigeria. The optimal cost for this schedule for the year 2015 is₦4.172 billion. Plant (machine) capacities of the nation's refineries should be adjusted to meet the projected production to satisfy the demand.

Finally, it is strongly recommended that proper data keeping and documentation by NNPC and PPMC should be strengthened for further research work.

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Appendix

```
LPSolve(60·x[11] + 229·x[12] + 426·x[13] + 979·x[14] + 531
    ·x[15] + 656·x[16] + 821·x[17] + 868·x[18] + 907·x[19]
    + 1093·x[110] + 687·x[111] + 775·x[112] + 1031·x[113]
    + 1320·x[114] + 1646·x[115] + 1095·x[116] + 1135·x[117]
    + 571·x[21] + 399·x[22] + 596·x[23] + 1149·x[24] + 99
    ·x[25] + 223·x[26] + 388·x[27] + 445·x[28] + 474·x[29]
    + 660·x[210] + 488·x[211] + 576·x[212] + 833·x[213] + 1123
    ·x[214] + 1447·x[215] + 896·x[216] + 936·x[217] + 792·x[31]
    + 622·x[32] + 818·x[33] + 1667·x[34] + 554·x[35] + 679
    ·x[36] + 844·x[37] + 890·x[38] + 930·x[39] + 1117·x[310]
    + 165·x[311] + 251·x[312] + 180·x[313] + 470·x[314] + 794
    ·x[315] + 244·x[316] + 284·x[317], {x[11] + x[12] + x[13]
    + x[14] + x[15] + x[16] + x[17] + x[18] + x[19] + x[110]
    + x[111] + x[112] + x[113] + x[114] + x[115] + x[116]
    + x[117] ≤ 7201849, x[21] + x[22] + x[23] + x[24] + x[25]
    + x[26] + x[27] + x[28] + x[29] + x[210] + x[211] + x[212]
    + x[213] + x[214] + x[215] + x[216] + x[217] ≤ 3192156,
    x[31] + x[32] + x[33] + x[34] + x[35] + x[36] + x[36]
    + x[37] + x[38] + x[39] + x[310] + x[311] + x[312] + x[313]
    + x[314] + x[315] + x[316] + x[317] ≤ 2603375, x[11]
    + x[21] + x[31] ≥ 788863, x[12] + x[22] + x[32] ≥ 950013,
    x[13] + x[23] + x[33] ≥ 435369, x[14] + x[24] + x[34]
    ≥ 154653, x[15] + x[25] + x[35] ≥ 596519, x[16] + x[26]
    + x[36] ≥ 244326, x[17] + x[27] + x[37] ≥ 1020195, x[18]
    + x[28] + x[38] ≥ 1846744, x[19] + x[29] + x[39]
    ≥ 1316504, x[110] + x[210] + x[310] ≥ 389883, x[111]
    + x[211] + x[311] ≥ 341797, x[112] + x[212] + x[312]
    ≥ 63681, x[113] + x[213] + x[313] ≥ 230031, x[114] + x[214]
    + x[314] ≥ 144256, x[115] + x[215] + x[315] ≥ 171461,
    x[116] + x[216] + x[316] ≥ 283315, x[117] + x[217] + x[317]
    ≥ 98771}, assume = {nonnegative, integer});
```

$$\big[4172475711, \big[x_{11} = 788863, x_{12} = 950013, x_{13} = 435369, x_{14} = 154653, x_{15} = 375271, x_{16} = 0, x_{17} = 0, x_{18} = 1846744, x_{19} = 0, x_{21} = 0, x_{22} = 0, x_{23} = 0, x_{24} = 0, x_{25} = 221248, x_{26} = 244326, x_{27} = 1020195, x_{28} = 0, x_{29} = 1316504, x_{31} = 0, x_{32} = 0, x_{33} = 0, x_{34} = 0, x_{35} = 0, x_{36} = 0, x_{37} = 0, x_{38} = 0, x_{39} = 0, x_{110} = 0, x_{111} = 0, x_{112} = 0, x_{113} = 0, x_{114} = 0, x_{115} = 0, x_{116} = 0, x_{117} = 0, x_{210} = 389883, x_{211} = 0, x_{212} = 0, x_{213} = 0, x_{214} = 0, x_{215} = 0, x_{216} = 0, x_{217} = 0, x_{310} = 0, x_{311} = 341797, x_{312} = 63681, x_{313} = 230031, x_{314} = 144256, x_{315} = 171461, x_{316} = 283315, x_{317} = 98771\big]\big]$$