MEASURING OF COEFFICIENT OF RESTITUTION WITH THE AID OF R-C CIRCUIT

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ABSTRACT

R-C circuits are useful where timing errors of a few percent are acceptable. The oscillator circuit used here is often referred to as the clock, and it is used here to generate time intervals that are counted to produce the desired timing. This research paper presents a simple method that allows the determination of the coefficient of restitution of bouncing balls using the R-C circuit as a timer, a meter rule and graphical analysis. The experiment emphasizes on the simple models and the vertical non-spinning motion of the balls between collisions which leads to good results. The average values of the coefficient of restitution of the rubber squash ball, plastic table tennis ball and rubber lawn tennis ball used in the experiment are 0.3122, 0.6223 and 0.7818 respectively.

KEYWORDS: Coefficient of Restitution, Bouncing Balls, RC Circuit, Graphical Analysis, Collisions.

INTRODUCTION

The coefficient of restitution (COR) is a measure of how much kinetic energy is lost in a real collision. It determines the nature of the collision as being perfectly elastic ($\varepsilon = 1$), perfectly inelastic ($\varepsilon = 0$) and real ($0 < \varepsilon < 1$).The COR is a convenient way of specifying how much energy in a collision is transferred to internal energy. It depends on the nature of the colliding materials but under practical conditions, it is a constant factor representing the energy transfer from kinetic energy to internal energy. Dropping a ball on a concrete surface from a given height is one of the simplest ways of determining the COR of the ball-surface material combination. A host of authors (Aguiar and Laudares, 2003; Stengaard and Lagsgaard, 2001; Smith *et al*,. 1981; Guercio and Zanetti, 1987; Bemstein, 1977; Cross, 1999; Foong *et al*, 2004; Bridge, 1998) have shown interest in the measurement of the COR for a collision between a ball and a flat concrete surface using a stopwatch or the sound produced by the impact/collision between the two surfaces. The standard procedure employed by these authors was allowing the ball to fall down vertically from a known height and make a large number of bounces/collisions with the concrete surface before coming to rest. For example, a recording of the sound of impact of the ball, each time, with the surface is made either with the aid of a pen recorder or computer sensors or high speed cameras and forces. The use of a stopwatch can be a source of error in the time interval measurement, as the successive bounces do not remain vertical and too much energy is transferred to the surface if the ball is allowed a large number of bounces. Our aim in this work is to describe a new technique for the measurement of the COR which is simpler, cheaper and overcomes the errors due to non-vertical bounces and too much loss of energy to the surface for large number of bounces.

Theory

Consider a ball of mass m dropped from initial height h on a horizontal hard surface. Assuming air resistance is negligible and only vertical motion is considered, using the law of conservation of energy, we can write

$$\frac{1}{2}mv^2 = mgh \qquad 1$$

where v is the velocity of the ball as it strikes the collision surface. From equation (1) the velocity of the ball as it strikes the surface is given by

$$v = \sqrt{2gh} \qquad 2$$

The coefficient of restitution ε for a given pair of colliding bodies for real collisions is the ratio of the velocity v_s of separation after a collision to the velocity v_a of approach before a collision given by

$$\varepsilon = \frac{v_s}{v_a}$$

Hence v_s can be obtained once v_a is known as the velocity of the ball changes to

$$v_s = \varepsilon v_a \qquad 3$$

After the first impact with the colliding surface,the ball again rises to a certain maximum height and falls down to suffer a second impact with the hard surface. The time interval between the first and second collisions is given by (Ajay, 2009)

$$t = \frac{2v_s}{g} \qquad 4$$

Substituting equation (3) in equation (4) yields

$$t = \frac{2\varepsilon v_a}{g} \qquad 5$$

It is worthy to note that the velocity of approach v_a just before the second impact/collision occurs is of the form of equation (2). Therefore, substituting $v_a = \sqrt{2gh}$ in equation (5) we have

$$t = \varepsilon\sqrt{\frac{8h}{g}} \qquad 6$$

This is the equation relating the time interval t between the first and second collisions, the initial height and the COR for the ball-surface combination.

Now we know that the initial voltage V_o across a charged capacitor of capacitance C decreases to V as it discharges through a resistor of resistance R in time t is given by

$$V = V_o e^{\frac{-t}{RC}} \qquad 7$$

Or $$t = RC\ln\left(\frac{V_0}{V}\right) \quad 8$$

If the series RC circuit is incorporated in the experimental-setup such that the time in equation (8) is equivalent to the time interval between two successive impacts which can be calculated. Thus, equation (**8**) is the expression for the time interval between the first and second impacts for the ball-metal surface combination under the influence of gravity.

The graph of the calculated t values from equation (8) against the square root of the initial height h according to equation (6) has a slope, $s = \varepsilon\sqrt{\frac{8}{g}}$ from which the average COR ε can be calculated.

This method has advantage over use of stopwatch to measure the small time interval taken by free falling objects through short distances. For example, the time t taken for a freely falling objects from rest at a height of 1 m according the formula $h = \frac{1}{2}gt^2$ is about 0.45 seconds neglecting air resistance and taking g = 9.81 m/s.

In practice it is difficult to measure such a small time by means of stop-watch because the reaction time of the experimenter to start and stop the stopwatch may be greater than the time interval of interest, Yerima *et al*, (2008).It is simpler than the cathode ray oscilloscope (CRO) technique (Ajay, 2009) since it does not require trained technicians for its operation. Its components are cheaper and available in our local markets unlike CRO which is sophisticated, expensive and not available in most of our schools.

Methodology

The method employed for determining the COR for the ball-metal surface combination is to allow the ball to fall freely from a known height *h*and make precisely two impacts with the flat, smooth, hard and massive metal surface below it. The major uncertainties in the experiment come from the release of the ball which may induce unwanted spinning and the quality of the horizontal surface on which the ball bounces. These uncertainties were avoided by turning the screw S gently to allow the ball fall freely without spinning on firmly fixed flat, hard, massive and smooth metal plate. The experimental set-up is shown in fig 1, in which RC circuit was used as a timer. In fig 1b, with relays K_1 and K_3 closed and k_2 opened the capacitor was charged through R to a maximum voltage V_o. With relay K_1 opened and the ball released from the rest position at the height h collided with the surface below, K_2 closed instantly and at the same time the capacitor started discharging. The ball bounces back and falls to collide for the second time with the surface. The relay K_3 opened and the discharging process stopped instantly the second collision occurred when the potential difference across the capacitor has dropped to a value V in agreement with equation (7a). The value of V across C after the time interval t between the first and second collisions was determined using a digital voltmeter. The time interval t between the first and second impacts was calculated using equation

(7b).The base of the metal plate was firmly fixed to the ground to avoid the ball undergo non-vertical bouncing or unwanted vibrations after first collision. The surface area of the metal plate was made small of radius 2 cm for golf/plastic balls and 3.5 cm for football. The smaller the area of the surface of the plate, the second collision on it proves that the ball bounces vertically after first collision. Thus, all bounces outside these radii were considered non-vertical and were neglected.

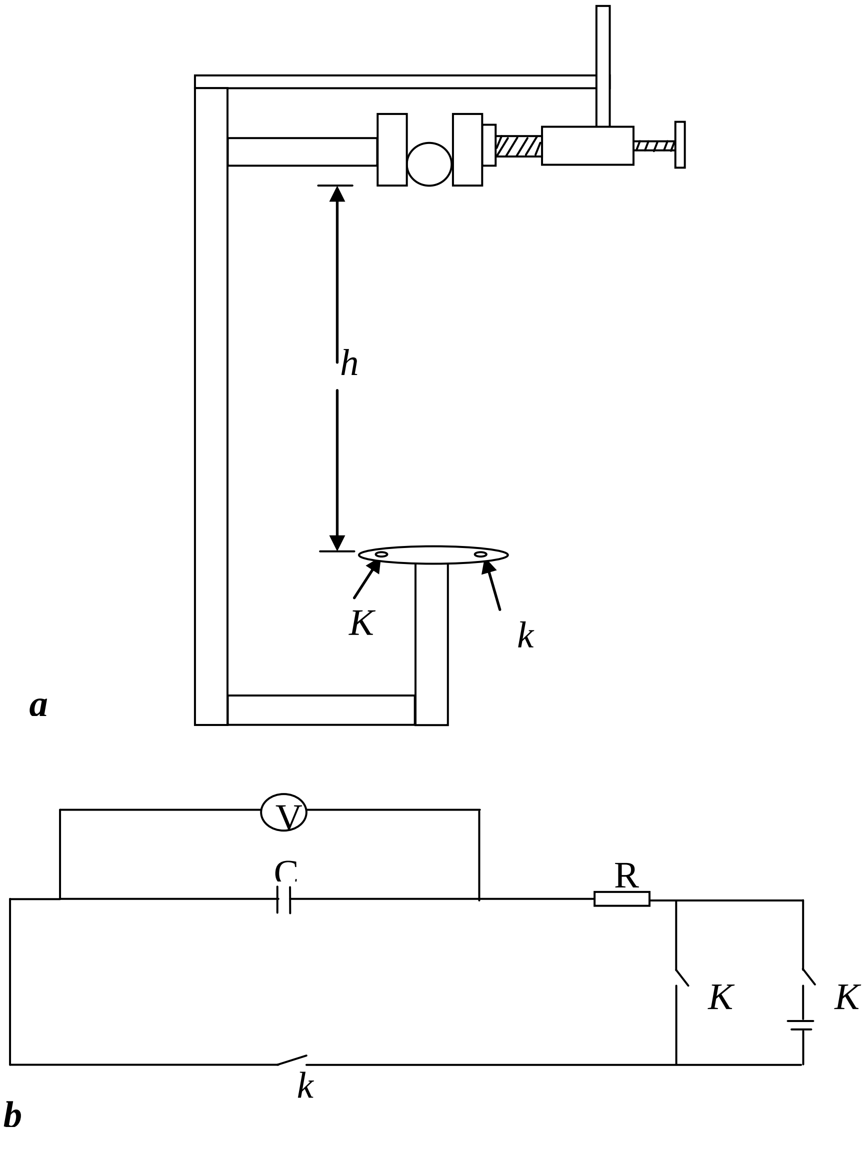

Figure **1** : (***a***)free fall of ball; (**b**) RC series circuit

RESULT AND DISCUSSION

Using equation **(8)** the values of the time intervals between two successive collisions of three different balls with the hard smooth metal surface below have been measured with the aid of an R-C circuit (Table 1). The results show that for a given height, *h* the time interval for the rubber lawn tennis ball is highest and least for rubber squash ball.

Table 1: Time intervals between successive collisions for ball-metal surface combinations

$\sqrt{h}$ (m)	Time, t (ms) Rubber squash ball	Plastic table tennis ball	Rubber lawn tennis ball
0.72	285.7	457.1	514.3
0.87	342.9	542.9	628.6
1.02	393.1	628.6	725.7
1.12	441.4	685.7	800.0
1.23	445.7	731.4	86.86
1.32	457.1	788.6	942.9
1.4	485.7	851.4	1000.0

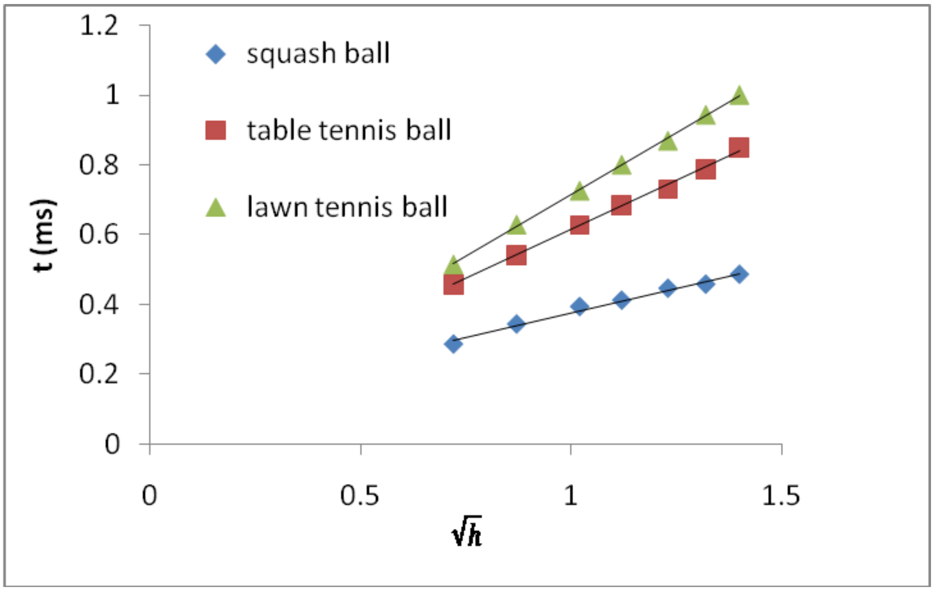

Figure:1 Time intervals between successive collisions for ball-metal surface

Fig. 1 shows the variation of the time interval between successive impacts with the initial height. Using least squares method the following empirical equations were obtained

$$t_{sb} = 0.706\sqrt{h} + 0.007 \quad 9$$

$$t_{tb} = 0.562\sqrt{h} + 0.051 \quad 10$$

$$t_{lb} = 0.282\sqrt{h} + 0.092 \quad 11$$

The slopes of these regression lines and the R^2 values from the computer-generated fits using excel are shown in table 2, along with the average values of COR determined from the slopes. Equations **9-11** clearly show that we have not forced the fits to pass through the origin, as would be expected from the model discussed above. Non-zero intercepts on the time-axis represent systematic errors that can be minimized by attentive experimenter. Even with these experimental details in mind, the experiment is simple and yields results that are consistent with expectations.

Table 2: Results from least squares method

Ball	Regression slope ($sm^{-1/2}$)	R^2 value	Average COR
Rubber squash ball	0.282	0.986	0.3122
Plastic table tennis ball	0.562	0.996	0.6223
Rubber lawn tennis ball	0.706	0.999	0.7818

The method is simple and cheap as it does not require sophisticated and expensive equipment such as CRO with amplifier used as sound detector (Ajay, 2009) that require trained technicians to operate it. It is also precise and accurate since it does not require us to measure the rebound height of the bouncing ball after any impact and no more than two collisions are involved in a single measurement.

CONCLUSION

The experiment was easily performed without the need for special equipment. It emphasizes on careful measurements and linear regression analysis, and uses a simple model to determine the COR of bouncing balls. The values of the COR for three different metal-ball surface have been determined using R-C circuit as a timer for comparative studies. This simple timer is based on the charging and discharging of a capacitor through a resistor. A timing circuit was used to time a short time delays of a few nanoseconds to accurately monitor times of descend of different balls through a vertical height, *h* reliably, accurately and repeatedly. The values of the time intervals between two successive impacts lie between 286 *m*s and 1000 *m*s and the average values of the COR lie between 0.3122 and 0.7818.

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